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Applications of Integrals

TOPIC 1 **Curve & X-axis Between two Ordinates, Area of the Region Bounded by a Curve & Y-axis Between two Abscissa**

1. The area (in sq. units) of the region $A = \{(x, y) : (x-1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$, where $[t]$ denotes the greatest integer function, is :
[Sep. 05, 2020 (II)]

(a) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (b) $\frac{4}{3}\sqrt{2} + 1$

(c) $\frac{8}{3}\sqrt{2} - 1$ (d) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$

2. The area (in sq. units) of the region

$\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, \frac{1}{2} \le x \le 2\}$ is :
[Sep. 03, 2020 (I)]

(a) $\frac{23}{16}$ (b) $\frac{79}{24}$ (c) $\frac{79}{16}$ (d) $\frac{23}{6}$

3. Given: $f(x) = \begin{cases} x & , \quad 0 \le x < \frac{1}{2} \\ \frac{1}{2} & , \quad x = \frac{1}{2} \\ 1-x & , \quad \frac{1}{2} < x \le 1 \end{cases}$

and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in R$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :
[Jan. 9, 2020 (II)]

(a) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (c) $\frac{1}{2} - \frac{\sqrt{3}}{4}$ (d) $\frac{1}{2} + \frac{\sqrt{3}}{4}$

4. The area (in sq. units) of the region
A = {(x, y) ∈R × R|0 d" x d"3, 0 d" y d" 4, y d" x^2 + 3x} is :
[April 8, 2019 (I)]

(a) $\frac{53}{6}$ (b) 8 (c) $\frac{59}{6}$ (d) $\frac{26}{3}$

5. The area of the region
$A = \{(x, y): 0 \le y \le x|x| + 1$ and $-1 \le x \le 1\}$ in sq. units is:
[Jan. 09, 2019 (II)]

(a) $\frac{2}{3}$ (b) 2 (c) $\frac{4}{3}$ (d) $\frac{1}{3}$

6. Let $g(x) = \cos x^2, f(x) = \sqrt{x}$, and $\alpha, \beta\ (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is : **[2018]**

(a) $\frac{1}{2}(\sqrt{3}+1)$ (b) $\frac{1}{2}(\sqrt{3}-\sqrt{2})$

(c) $\frac{1}{2}(\sqrt{2}-1)$ (d) $\frac{1}{2}(\sqrt{3}-1)$

7. Let $f : [-2, 3] \to [0, \infty)$ be a continuous function such that $f(1-x) = f(x)$ for all $x \in [-2, 3]$.
If R_1 is the numerical value of the area of the region bounded by $y = f(x)$, $x = -2$, $x = 3$ and the axis of x and

$R_2 = \int_{-2}^{3} x f(x)\,dx$, then : **[Online April 25, 2013]**

(a) $3R_1 = 2R_2$ (b) $2R_1 = 3R_2$
(c) $R_1 = R_2$ (d) $R_1 = 2R_2$

8. Let $f(x)$ be a non – negative continuous function such that the area bounded by the curve $y = f(x)$, x - axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is

$\left(\beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is **[2005]**

(a) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (b) $\left(\frac{\pi}{4}-\sqrt{2}+1\right)$

(c) $\left(1-\frac{\pi}{4}-\sqrt{2}\right)$ (d) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

9. The area enclosed between the curve $y=\log_e(x+e)$ and the coordinate axes is **[2005]**

(a) 1 (b) 2 (c) 3 (d) 4

10. If $y=f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of 3/4 square unit with the axes then $\int_0^2 xf'(x)dx$ is **[2002]**

(a) 3/2 (b) 1 (c) 5/4 (d) –3/4

TOPIC 2 Different Cases of Area Bounded Between the Curves

11. The area (in sq. units) of the region $A=\{(x,y):|x|+|y|\le 1, 2y^2\ge |x|\}$ is: **[Sep. 06, 2020 (I)]**

(a) $\frac{1}{3}$ (b) $\frac{7}{6}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$

12. The area (in sq. units) of the region enclosed by the curves $y=x^2-1$ and $y=1-x^2$ is equal to: **[Sep. 06, 2020 (II)]**

(a) $\frac{4}{3}$ (b) $\frac{8}{3}$ (c) $\frac{7}{2}$ (d) $\frac{16}{3}$

13. Consider a region $R=\{(x,y)\in\mathbf{R}^2 : x^2\le y\le 2x\}$. If a line $y=\alpha$ divides the area of region R into two equal parts, then which of the following is **true**? **[Sep. 02, 2020 (II)]**

(a) $\alpha^3-6\alpha^2+16=0$ (b) $3\alpha^2-8\alpha^{3/2}+8=0$

(c) $3\alpha^2-8\alpha+8=0$ (d) $\alpha^3-6\alpha^{3/2}-16=0$

14. The area of the region, enclosed by the circle $x^2+y^2=2$ which is not common to the region bounded by the parabola $y^2=x$ and the straight line $y=x$, is: **[Jan. 7, 2020 (I)]**

(a) $(24\pi-1)$ (b) $(6\pi-1)$

(c) $(12\pi-1)$ (d) $(12\pi-1)/6$

15. The area (in sq. units) of the region $\{(x,y)\in R^2 | 4x^2\le y\le 8x+12\}$ is: **[Jan. 7, 2020 (II)]**

(a) $\frac{125}{3}$ (b) $\frac{128}{3}$ (c) $\frac{124}{3}$ (d) $\frac{127}{3}$

16. For $a>0$, let the curves $C_1: y^2=ax$ and $C_2: x^2=ay$ intersect at origin O and a point P. Let the line $x=b$ $(0<b<a)$ intersect the chord OP and the x-axis at points Q and R, respectively. If the line $x=b$ bisects the area bounded by the curves, C_1 and C_2, and the area of $\Delta OQR=\frac{1}{2}$, then

(a) $x^6-6x^3+4=0$ (b) $x^6-12x^3+4=0$

(c) $x^6+6x^3-4=0$ (d) $x^6-12x^3-4=0$

17. The area (in sq. units) of the region $\{(x,y)\in R^2: x^2\le y\le |3-2x|\}$, is: **[Jan. 8, 2020 (II)]**

(a) $\frac{32}{3}$ (b) $\frac{34}{3}$ (c) $\frac{29}{3}$ (d) $\frac{31}{3}$

18. If the area (in sq. units) of the region $\{(x,y): y^2\le 4x, x+y\le 1, x\ge 0, y\ge 0\}$ is $a\sqrt{2}+b$, then $a-b$ is equal to : **[April 12, 2019 (I)]**

(a) $\frac{10}{3}$ (b) 6 (c) $\frac{8}{3}$ (d) $-\frac{2}{3}$

19. If the area (in sq. units) bounded by the parabola $y^2=4\lambda x$ and the line $y=\lambda x$, $\lambda>0$, is $\frac{1}{9}$, then λ is equal to : **[April 12, 2019 (II)]**

(a) $2\sqrt{6}$ (b) 48 (c) 24 (d) $4\sqrt{3}$

20. The region represented by $|x-y|\le 2$ and $|x+y|\le 2$ is bounded by a : **[April 10, 2019 (I)]**

(a) square of side length $2\sqrt{2}$ units

(b) rhombus of side length 2 units

(c) square of area 16 sq. units

(d) rhombus of area $8\sqrt{2}$ sq. units

21. The area (in sq. units) of the region bounded by the curves $y=2^x$ and $y=|x+1|$, in the first quadrant is : **[April 10, 2019 (II)]**

(a) $\log_e 2+\frac{3}{2}$ (b) $\frac{3}{2}$

(c) $\frac{1}{2}$ (d) $\frac{3}{2}-\frac{1}{\log_e 2}$

22. The area (in sq. units) of the region $A=\{(x,y): x^2\le y\le x+2\}$ is: **[April 9, 2019 (I)]**

(a) $\frac{10}{3}$ (b) $\frac{9}{2}$ (c) $\frac{31}{6}$ (d) $\frac{13}{6}$

23. The area (in sq. units) of the region $A=\{(x,y):\frac{y^2}{2}\le x\le y+4\}$ is: **[April 09, 2019 (II)]**

(a) $\frac{53}{3}$ (b) 30 (c) 16 (d) 18

24. Let $S(\alpha)=\{(x,y): y^2\le x, 0\le x\le\alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ, $0<\lambda<4$, $A(\lambda):A(\alpha)=2:5$, then λ equals : **[April 08, 2019 (II)]**

(a) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (b) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$

(c) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (d) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

25. The area (in sq. units) of the region bounded by the parabola, $y=x^2+2$ and the lines, $y=x+1$, $x=0$ and $x=3$, is : **[Jan. 12, 2019 (I)]**

(a) $\frac{15}{4}$ (b) $\frac{21}{2}$ (c) $\frac{17}{4}$ (d) $\frac{15}{2}$

26. The area (in sq. units) of the region bounded by the curve $x^2=4y$ and the straight line $x=4y-2$ is : **[Jan. 11, 2019 (I)]**

(a) $\frac{5}{4}$ (b) $\frac{9}{8}$ (c) $\frac{7}{8}$ (d) $\frac{3}{4}$

27. The area (in sq. units) in the first quadrant bounded by the parabola, $y=x^2+1$, the tangent to it at the point (2, 5) and the coordinate axes is : **[Jan. 11, 2019 (II)]**

(a) $\frac{8}{3}$ (b) $\frac{37}{24}$ (c) $\frac{187}{24}$ (d) $\frac{14}{3}$

28. If the area enclosed between the curves $y=kx^2$ and $x=ky^2$, $(k>0)$, is 1 square unit. Then k is: **[Jan. 10, 2019 (I)]**

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$

29. The area (in sq. units) bounded by the parabola $y=x^2-1$, the tangent at the point (2, 3) to it and the y-axis is: **[Jan. 9, 2019 (I)]**

(a) $\frac{8}{3}$ (b) $\frac{32}{3}$ (c) $\frac{56}{3}$ (d) $\frac{14}{3}$

30. If the area of the region bounded by the curves, $y=x^2$, $y=\frac{1}{x}$ and the lines $y=0$ and $x=t\,(t>1)$ is 1 sq. unit, then t is equal to **[Online April 16, 2018]**

(a) $\frac{4}{3}$ (b) $e^{2/3}$ (c) $\frac{3}{2}$ (d) $e^{3/2}$

31. The area (in sq. units) of the region

$\{x \in R : x \geq 0, y \geq 0, y \geq x-2 \text{ and } y \leq \sqrt{x}\}$, is **[Online April 15, 2018]**

(a) $\frac{13}{3}$ (b) $\frac{10}{3}$ (c) $\frac{5}{3}$ (d) $\frac{8}{3}$

32. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x+y \leq 3, x^2 \leq 4y \text{ and } y \leq 1+\sqrt{x}\}$ is : **[2017]**

(a) $\frac{5}{2}$ (b) $\frac{59}{12}$ (c) $\frac{3}{2}$ (d) $\frac{7}{3}$

33. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2+y^2=4$ and $y^2=3x$, is : **[Online April 8, 2017]**

(a) $\frac{1}{2\sqrt{3}}+\frac{\pi}{3}$ (b) $\frac{1}{\sqrt{3}}+\frac{2\pi}{3}$

(c) $\frac{1}{2\sqrt{3}}+\frac{2\pi}{3}$ (d) $\frac{1}{\sqrt{3}}+\frac{4\pi}{3}$

34. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2+y^2 \leq 4x, x \geq 0, y \geq 0\}$ is : **[2016]**

(a) $\pi-\frac{4\sqrt{2}}{3}$ (b) $\frac{\pi}{2}-\frac{2\sqrt{2}}{3}$

(c) $\pi-\frac{4}{3}$ (d) $\pi-\frac{8}{3}$

35. The area (in sq. units) of the region described by $A=\{(x, y) \mid y \geq x^2-5x+4, x+y \geq 1, y \leq 0\}$ is: **[Online April 9, 2016]**

(a) $\frac{19}{6}$ (b) $\frac{17}{6}$ (c) $\frac{7}{2}$ (d) $\frac{13}{6}$

36. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x-1\}$ is **[2015]**

(a) $\frac{15}{64}$ (b) $\frac{9}{32}$ (c) $\frac{7}{32}$ (d) $\frac{5}{64}$

37. The area (in square units) of the region bounded by the curves $y+2x^2=0$ and $y+3x^2=1$, is equal to : **[Online April 10, 2015]**

(a) $\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

38. The area of the region described by

$A=\{(x, y) : x^2+y^2 \leq 1 \text{ and } y^2 \leq 1-x\}$ is: **[2014]**

(a) $\frac{\pi}{2}-\frac{2}{3}$ (b) $\frac{\pi}{2}+\frac{2}{3}$ (c) $\frac{\pi}{2}+\frac{4}{3}$ (d) $\frac{\pi}{2}-\frac{4}{3}$

39. The area of the region above the x-axis bounded by the curve $y=\tan x$, $0 \leq x \leq \frac{\pi}{2}$ and the tangent to the curve at $x=\frac{\pi}{4}$ is: **[Online April 19, 2014]**

(a) $\frac{1}{2}\left(\log 2-\frac{1}{2}\right)$ (b) $\frac{1}{2}\left(\log 2+\frac{1}{2}\right)$

(c) $\frac{1}{2}(1-\log 2)$ (d) $\frac{1}{2}(1+\log 2)$

40. Let $A=\{(x, y): y^2 \leq 4x, y-2x \geq -4\}$. The area (in square units) of the region A is: **[Online April 9, 2014]**

(a) 8 (b) 9 (c) 10 (d) 11

41. The area (in square units) bounded by the curves $y=\sqrt{x}$, $2y-x+3=0$, x-axis, and lying in the first quadrant is : **[2013]**

(a) 9 (b) 36 (c) 18 (d) $\frac{27}{4}$

42. The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$, and above x-axis is : **[Online April 23, 2013]**

(a) $2\sqrt{2}$ (b) $2\sqrt{2}-2$
(c) $2\sqrt{2}+2$ (d) 0

43. The area of the region (in sq. units), in the first quadrant bounded by the parabola $y = 9x^2$ and the lines $x = 0, y = 1$ and $y = 4$, is : **[Online April 22, 2013]**

(a) 7/9 (b) 14/3 (c) 7/3 (d) 14/9

44. The area bounded by the curve $y = \ln(x)$ and the lines $y = 0, y = \ln(c)$ and $x = 0$ is equal to : **[Online April 9, 2013]**

(a) 3 (b) $3\ln(c) - 2$
(c) $3\ln(c) + 2$ (d) 2

45. The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is : **[2012]**

(a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$ (c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$

46. The area bounded by the parabola $y^2 = 4x$ and the line $2x - 3y + 4 = 0$, in square unit, is **[Online May 26, 2012]**

(a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{1}{2}$

47. The area of the region bounded by the curve $y = x^3$, and the lines, $y = 8$, and $x = 0$, is **[Online May 19, 2012]**

(a) 8 (b) 12 (c) 10 (d) 16

48. If a straight line $y - x = 2$ divides the region $x^2 + y^2 \le 4$ into two parts, then the ratio of the area of the smaller part to the area of the greater part is **[Online May 12, 2012]**

(a) $3\pi - 8 : \pi + 8$ (b) $\pi - 3 : 3\pi + 3$
(c) $3\pi - 4 : \pi + 4$ (d) $\pi - 2 : 3\pi + 2$

49. The area enclosed by the curves $y = x^2$, $y = x^3$, $x = 0$ and $x = p$, where $p > 1$, is 1/6. The p equals **[Online May 12, 2012]**

(a) 8/3 (b) 16/3 (c) 2 (d) 4/3

50. The parabola $y^2 = x$ divides the circle $x^2 + y^2 = 2$ into two parts whose areas are in the ratio **[Online May 7, 2012]**

(a) $9\pi + 2 : 3\pi - 2$ (b) $9\pi - 2 : 3\pi + 2$
(c) $7\pi - 2 : 2\pi - 3$ (d) $7\pi + 2 : 3\pi + 2$

51. The area bounded by the curves **[2011 RS]**

$y^2 = 4x$ and $x^2 = 4y$ is:

(a) $\frac{32}{3}$ sq units (b) $\frac{16}{3}$ sq units
(c) $\frac{8}{3}$ sq. units (d) 0 sq. units

52. The area of the region enclosed by the curves $y = x, x = e, y = \frac{1}{x}$ and the positive x-axis is **[2011]**

(a) 1 square unit (b) $\frac{3}{2}$ square units
(c) $\frac{5}{2}$ square units (d) $\frac{1}{2}$ square unit

53. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is **[2010]**

(a) $4\sqrt{2}+2$ (b) $4\sqrt{2}-1$
(c) $4\sqrt{2}+1$ (d) $4\sqrt{2}-2$

54. The area of the region bounded by the parabola $(y-2)^2 = x - 1$, the tangent of the parabola at the point (2, 3) and the x-axis is: **[2009]**

(a) 6 (b) 9 (c) 12 (d) 3

55. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to **[2008]**

(a) $\frac{5}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

56. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[2007]**

(a) 1/6 (b) 1/3 (c) 2/3 (d) 1

57. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4, y = 4$ and the coordinate axes. If S_1, S_2, S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is **[2005]**

(a) 1 : 2 : 1 (b) 1 : 2 : 3 (c) 2 : 1 : 2 (d) 1 : 1 : 1

58. The area of the region bounded by the curves $y = |x - 2|, x = 1, x = 3$ and the x-axis is **[2004]**

(a) 4 (b) 2 (c) 3 (d) 1

59. The area of the region bounded by the curves $y = |x - 1|$ *and* $y = 3 - |x|$ is **[2003]**

(a) 6 sq. units (b) 2 sq. units
(c) 3 sq. units (d) 4 sq. units.

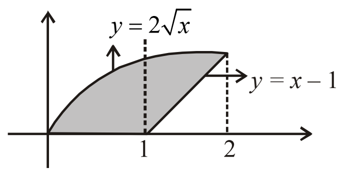
60. The area bounded by the curves $y = \ln x, y = \ln|x|, y = |\ln x|$ and $y = |\ln|x||$ is **[2002]**

(a) 4sq. units (b) 6 sq. units
(c) 10 sq. units (d) none of these

Hints & Solutions

1. **(a)** $[x]=0$ when $x\in[0,1)$ and $[x]=1$ when $x\in[1,2)$

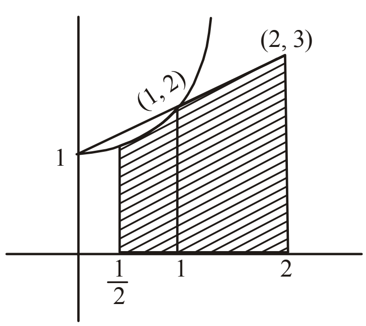
$$y=\begin{cases}0 & 0\le x<1\\ x-1 & 1\le x<2\end{cases}$$



$$\therefore A=\int_0^2 2\sqrt{x}\,dx-\frac{1}{2}(1)(1)$$

$$=\left.\frac{4x^{3/2}}{3}\right|_0^2-\frac{1}{2}=\frac{8\sqrt{2}}{3}-\frac{1}{2}$$

2. **(b)**



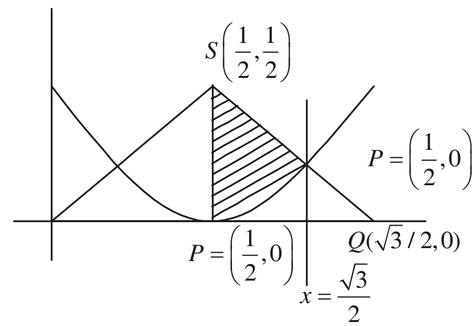
$$\text{Required area}=\int_{\frac{1}{2}}^{1}(x^2+1)dx+\int_1^2(x+1)dx$$

$$=\left[\frac{x^3}{3}+x\right]_{\frac{1}{2}}^{1}+\left[\frac{x^2}{2}+x\right]_1^2$$

$$=\left[\frac{4}{3}-\frac{13}{24}\right]+\frac{5}{2}=\frac{79}{24}.$$

3. **(b)** Coordinates of $P\left(\frac{1}{2},0\right),Q\left(\frac{\sqrt{3}}{2},0\right),R\left(\frac{\sqrt{3}}{2},1-\frac{\sqrt{3}}{2}\right)$

and $S\left(\frac{1}{2},\frac{1}{2}\right)$



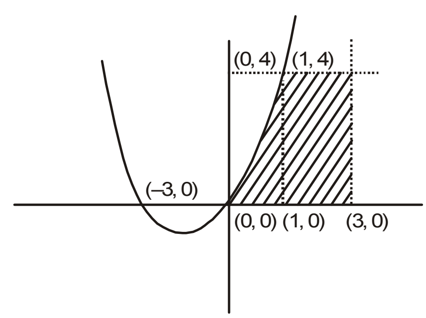
Required area = Area of trapezium $PQRS$

$$-\int_{1/2}^{\sqrt{3}/2}\left(x-\frac{1}{2}\right)^2dx$$

$$=\frac{1}{2}\left(\frac{\sqrt{3}-1}{2}\right)\left(\frac{1}{2}+1-\frac{\sqrt{3}}{2}\right)-\frac{1}{3}\left(\left(x-\frac{1}{2}\right)^3\right)_{1/2}^{\sqrt{3}/2}$$

$$=\frac{\sqrt{3}}{4}-\frac{1}{3}$$

4. **(c)** Since, the relation $y\le x^2+3x$ represents the region below the parabola in the 1st quadrant



$\because y=4$

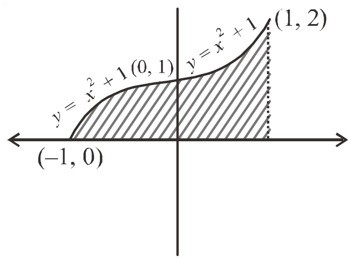
$\Rightarrow x^2+3x=4\Rightarrow x=1,-4$

$\therefore$ the required area = area of shaded region

$$=\int_0^1(x^2+3x)dx+\int_1^3 4.dx=\left[\frac{x^3}{3}+\frac{3x^2}{2}\right]_0^1+[4x]_1^3$$

$$=\frac{1}{3}+\frac{3}{2}+8=\frac{59}{6}$$

5. **(b)** Given $A = \{(x, y) : 0 \le y \le x|x| + 1 \text{ and} -1 \le x \le 1\}$



$\therefore$ Area of shaded region

$$= \int_{-1}^{0}(-x^2+1)dx + \int_{0}^{1}(x^2+1)dx$$

$$= \left(-\frac{x^3}{3}+x\right)_{-1}^{0} + \left(\frac{x^3}{3}+x\right)_{0}^{1}$$

$$= 0 - \left(\frac{1}{3}-1\right) + \left(\frac{1}{3}+1\right) - (0+0)$$

$$= \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2 \text{ square units}$$

6. **(d)** Here, $18x^2 - 9\pi x + \pi^2 = 0$

$\Rightarrow (3x - \pi)(6x - \pi) = 0$

$\Rightarrow \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

Also, $gof(x) = \cos x$

$$\therefore \text{ Req. area} = \int_{\pi/6}^{\pi/3} \cos x\,dx = \frac{\sqrt{3}-1}{2}$$

7. **(d)** We have

$$R_2 = \int_{-2}^{3} x\, f(x)\, dx = \int_{-2}^{3} (1-x)\, f(1-x)\, dx$$

$$\left[\text{Using} \int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx\right]$$

$$\Rightarrow R_2 = \int_{-2}^{3} (1-x)\, f(x)\, dx$$

$(\because f(x) = f(1-x) \text{ on } [-2, 3])$

$$\therefore R_2 + R_2 = \int_{-2}^{3} x\, f(x)\, dx + \int_{-2}^{3} (1-x)\, f(x)\, dx$$

$$= \int_{-2}^{3} f(x)\, dx = R_1$$

$\Rightarrow 2R_2 = R_1$

8. **(d)** From given condition

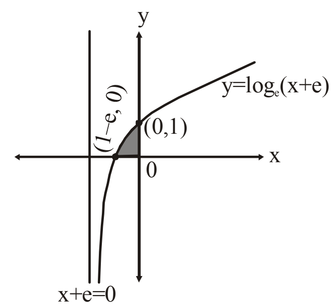
$$\int_{\pi/4}^{\beta} f(x)dx = \beta \sin\beta + \frac{\pi}{4}\cos\beta + \sqrt{2}\beta$$

Differentiating w. r . t β, we get

$$f(\beta) = \beta\cos\beta + \sin\beta - \frac{\pi}{4}\sin\beta + \sqrt{2}$$

$$f\left(\frac{\pi}{2}\right) = \beta\cdot 0 + \left(1-\frac{\pi}{4}\right)\sin\frac{\pi}{2} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$$

9. **(a)**



$$\text{Required area } A = \int_{1-e}^{0} y\,dx = \int_{1-e}^{0} \log_e(x+e)dx$$

put $x + e = t \Rightarrow dx = dt$ also when $x = 1 - e, t = 1$ and when $x = 0, t = e$

$$\therefore A = \int_{1}^{e} \log_e t\,dt = [t\log_e t - t]_1^e$$

$e - e - 0 + 1 = 1$

Hence the required area is 1 square unit.

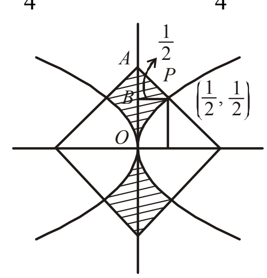
10. **(d)** Given that $\int_0^2 f(x)dx = \frac{3}{4}$; Now,

$$\int_0^2 xf'(x)dx = x\int_0^2 f'(x)dx - \int_0^2 f(x)dx$$

$$= [x\ f(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4}$$

$$= 0 - \frac{3}{4}\ (\because f(2) = 0) = -\frac{3}{4}.$$

11. **(d)**

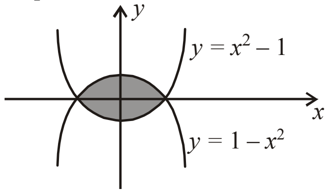


$$\text{Required area} = 4\left[\int_0^{\frac{1}{2}} 2y^2dy + \frac{1}{2}\text{area}(\Delta PAB)\right]$$

$$= 4\left[\frac{2}{3}\left[y^3\right]_0^{\frac{1}{2}} + \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right] = 4\left[\frac{2}{3}\times\frac{1}{8}+\frac{1}{8}\right]$$

$$= 4\times\frac{5}{24} = \frac{5}{6}.$$

12. **(b)** Required area

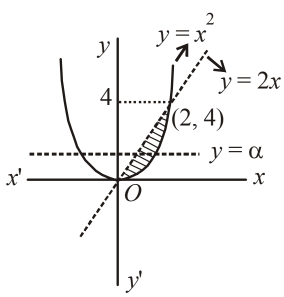


$$\text{Area } = 2\int_0^1 \left((1-x^2)-(x^2-1)\right)dx$$

$$= 4\int_0^1 (1-x^2)\,dx$$

$$= 4\left(x-\frac{x^3}{3}\right)\Bigg|_0^1 = 4\left(1-\frac{1}{3}\right) = 4\cdot\frac{2}{3} = \frac{8}{3} \text{ sq. units}$$

13. **(b)** Let $y = x^2$ and $y = 2x$



According to question

$$\therefore \int_0^{\alpha}\left(\sqrt{y}-\frac{y}{2}\right)dy = \int_{\alpha}^{4}\left(\sqrt{y}-\frac{y}{2}\right)dy$$

$$\Rightarrow \left[\frac{y^{3/2}}{\frac{3}{2}}\right]_0^{\alpha} - \left[\frac{y^2}{4}\right]_0^{\alpha} = \left[\frac{y^{3/2}}{\frac{3}{2}}\right]_{\alpha}^{4} - \left[\frac{y^2}{4}\right]_{\alpha}^{4}$$

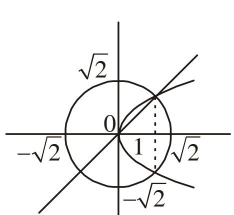
$$\Rightarrow \frac{2}{3}\alpha^{3/2} - \frac{\alpha^2}{4} = \frac{2}{3}(8-\alpha^{3/2}) - \frac{1}{4}(16-\alpha^2)$$

$$\Rightarrow \frac{4}{3}\alpha^{3/2} - \frac{\alpha^2}{2} = \frac{4}{3}$$

$$\Rightarrow 8\alpha^{3/2} - 3\alpha^2 = 8$$

$$\therefore 3\alpha^2 - 8\alpha^{3/2} + 8 = 0$$

14. **(d)** Total area – enclosed area between line and parabola

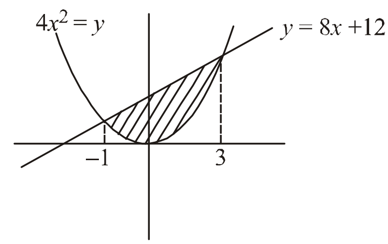


$$= 2\pi - \int_0^1 \sqrt{x} - x\,dx$$

$$= 2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2}\right)_0^1$$

$$= 2\pi - \left(\frac{2}{3}-\frac{1}{2}\right) = 2\pi - \left(\frac{1}{6}\right) = \frac{12\pi - 1}{6}$$

15. **(b)**



Given curves are

$4x^2 = y$ ...(i)

$y = 8x + 12$ ...(ii)

From eqns. (i) and (ii),

$4x^2 = 8x + 12$

$\Rightarrow x^2 - x - 3 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow x^2 - 3x + x - 3 = 0$

$\Rightarrow (x+1)(x-3) = 0$

$\Rightarrow x = -1, 3$

Required area bounded by curves is given by

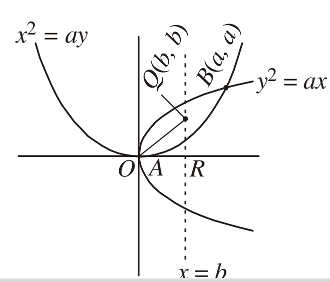
$$A = \int_{-1}^{3} (8x + 12 - 4x^2)dx$$

$$A = \frac{8x^2}{2} + 12x - \frac{4x^3}{3}\Bigg|_{-1}^{3}$$

$$= (4(9) + 36 - 36) - \left(4 - 12 + \frac{4}{3}\right)$$

$$= 36 + 8 - \frac{4}{3} = 44 - \frac{4}{3} = \frac{132-4}{3} = \frac{128}{3}$$

16. **(b)** Given eqns. are, $x^2 = ay$ and $y^2 = ax$



After solving, we get $x = a, y = a$
Now, coordinates of B is (a, a) and A is (0, 0)
Now, coordinates of Q is (b, b)

$\therefore \quad \frac{1}{2}b^2 = \frac{1}{2} \quad \Rightarrow \quad b = 1$

Area bounded by curves and $x = 1$ is

$$\int_0^1 \left(\sqrt{a}x^{1/2} - \frac{x^2}{a}\right)dx = \frac{1}{2}\int_0^a \left(\sqrt{a}x^{1/2} - \frac{x^2}{a}\right)dx$$

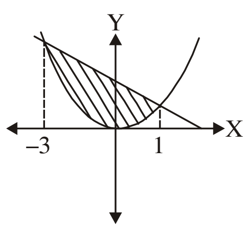
$\Rightarrow \quad \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$

$\Rightarrow \quad 4a\sqrt{a} - 2 = a^3$

$\Rightarrow \quad a^6 + 4a^3 + 4 = 16a^3$

$\Rightarrow \quad a^6 - 12a^3 + 4 = 0$

17. **(a)** Point of intersection of $y = x^2$ and $y = -2x + 3$ is obtained by $x^2 + 2x - 3 = 0$



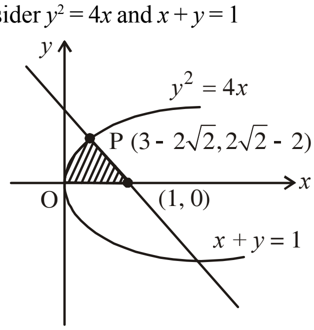
$\Rightarrow \quad x = -3, 1$

So, required area $= \int_{-3}^{1} (\text{line} - \text{parabola})dz$

$= \int_{-3}^{1} (3 - 2x - x^2)dx \quad = \left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^{1}$

$= (3)4 - 2\left(\frac{1^2 - 3^2}{2}\right) - \left(\frac{1^3 + 3^3}{3}\right) = 12 + 8 - \frac{28}{3} = \frac{32}{3}$

18. **(b)** Consider $y^2 = 4x$ and $x + y = 1$



Substituting $x = 1 - y$ in the equation of parabola,
$y^2 = 4(1 - y) \Rightarrow y^2 + 4y - 4 = 0$

$\Rightarrow \quad (y + 2)^2 = 8 \Rightarrow y + 2 = \pm 2\sqrt{2}$

Hence, required area

$= \int_0^{3-2\sqrt{2}} 2\sqrt{x}\,dx + \frac{1}{2} \times (2\sqrt{2} - 2) \times (2\sqrt{2} - 2)$

$= \left[2 \times \frac{2}{3}x^{3/2}\right]_0^{3-2\sqrt{2}} + \frac{1}{2}(8 + 4 - 8\sqrt{2})$

$= \frac{4}{3} \times (3 - 2\sqrt{2})\sqrt{3 - 2\sqrt{2}} + 6 - 4\sqrt{2}$

$= \frac{4}{3}(3 - 2\sqrt{2})(\sqrt{2} - 1) + 6 - 4\sqrt{2}$

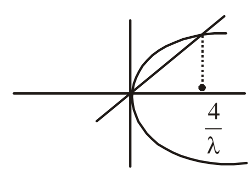
$[\because (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}]$

$= \frac{4}{3}(3\sqrt{2} - 3 - 4 + 2\sqrt{2}) + 6 - 4\sqrt{2}$

$= -\frac{10}{3} + \frac{8}{3}\sqrt{2} = a\sqrt{2} + b$

$\therefore \quad a = 8/3$ and $b = -10/3 \Rightarrow a - b = \frac{10}{3} + \frac{8}{3} = 6$

19. **(c)** Given parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$



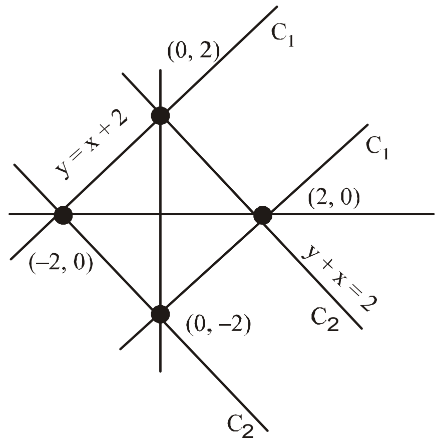
Putting $y = \lambda$ in $y^2 = 4\lambda x$, we get $x = 0, \frac{4}{\lambda}$

$\therefore \quad$ required area $= \int_0^{\frac{4}{\lambda}} (2\sqrt{\lambda x} - \lambda x)dx$

$= \left.\frac{2\sqrt{\lambda}.x^{3/2}}{3/2} - \frac{\lambda x^2}{2}\right|_0^{4/\lambda} = \frac{32}{3\lambda} - \frac{8}{\lambda}$

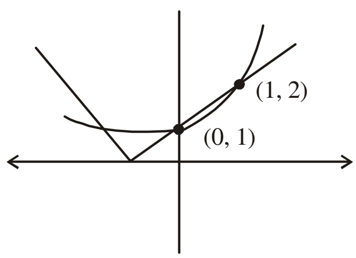
$= \frac{8}{3\lambda} = \frac{1}{9} \Rightarrow \lambda = 24$

20. **(a)** Let, $C_1 : |y - x| \le 2$
$C_2 : |y + x| \le 2$
By the diagram, region is square



Now, length of side $= \sqrt{2^2 + 2^2} = 2\sqrt{2}$

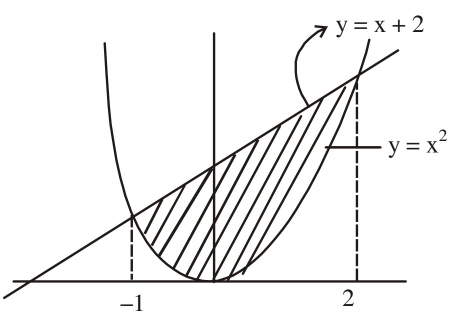
21. (d)



$$\text{Area} = \int_0^1 ((x+1) - 2^x)dx \quad \left(\because \text{Area} = \int y\,dx\right)$$

$$= \left[\frac{x^2}{2} + x - \frac{2^x}{\ln 2}\right]_0^1 = \left(\frac{1}{2} + 1 - \frac{2}{\ln 2}\right) - \left(\frac{-1}{\ln 2}\right) = \frac{3}{2} - \frac{1}{\ln 2}$$

22. (b)

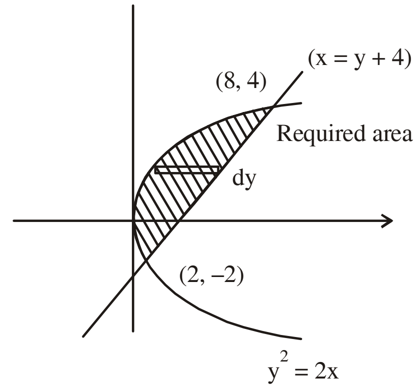


Required area is equal to the area under the curves $y \geq x^2$ and y d” x + 2

$$\therefore \text{ required area } \int_{-1}^{2} ((x+2) - x^2)dx$$

$$= \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)_{-1}^{2} = \left(2 + 4 - \frac{8}{3}\right) - \left(+\frac{1}{2} - 2 + \frac{1}{3}\right) = \frac{9}{2}$$

23. (d)



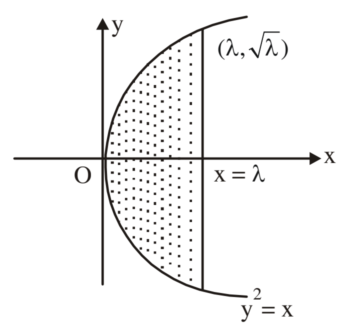
Given region, $A = \left\{(x, y) : \frac{y^2}{2} \leq x \leq y + 4\right\}$

Hence, area $= \int_{-2}^{4} x\,dy = \int_{-2}^{4} \left(y + 4 - \frac{y^2}{2}\right)dy$

$$= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4} = \left(8 + 16 - \frac{64}{6}\right) - \left(2 - 8 + \frac{8}{6}\right)$$

$$= \left(24 - \frac{32}{3}\right) - \left(-6 + \frac{4}{3}\right) = \frac{40}{3} + \frac{14}{3} = \frac{54}{3} = 18$$

24. (d)



Area of the region $= 2 \times \int_0^{\lambda} y\,dx = 2\int_0^{\lambda} \sqrt{x}\,dx$

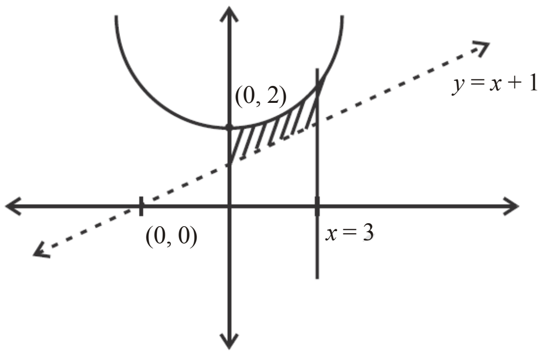
$$= 2 \times \frac{2}{3} \pi^{\frac{3}{2}}$$

$$A(\lambda) = 2 \times \frac{2}{3}(\lambda \times \sqrt{\lambda}) = \frac{4}{3}\lambda^{\frac{3}{2}}$$

Given, $\frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{\frac{3}{2}}}{8} = \frac{2}{5}$

$$\lambda = \left(\frac{16}{5}\right)^{\frac{2}{3}} = 4.\left(\frac{4}{25}\right)^{\frac{1}{3}}$$

25. (d)

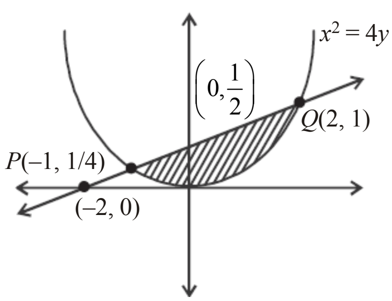


Area of the bounded region $\int_0^3 [(x^2+2)-(x+1)]dx$

$= \left[\frac{x^3}{3}-\frac{x^2}{2}+x\right]_0^3$

$= 9-\frac{9}{2}+3=\frac{15}{2}$

26. (b)



Let points of intersection of the curve and the line be P and Q

$x^2=4\left(\frac{x+2}{4}\right)$

$x^2-x-2=0$

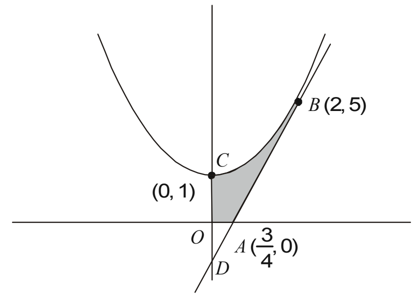
$x=2,-1$

Point are (2, 1) and $\left(-1,\frac{1}{4}\right)$

$\text{Area}=\int_{-1}^{2}\left[\left(\frac{x+2}{4}\right)-\left(\frac{x^2}{4}\right)\right]dx=\left[\frac{x^2}{8}+\frac{1}{2}x-\frac{x^3}{12}\right]_{-1}^{2}$

$=\left(\frac{1}{2}+1-\frac{2}{3}\right)-\left(\frac{1}{8}-\frac{1}{2}+\frac{1}{12}\right)=\frac{9}{8}$

27. (b)



The equation of parabola $x^2=y-1$

The equation of tangent at (2, 5) to parabola is

$y-5=\left(\frac{dy}{dx}\right)_{(2,5)}(x-2)$

$y-5=4(x-2)$

$4x-y=3$

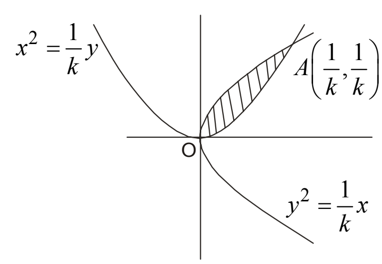
Then, the required area

$=\int_0^2\{(x^2+1)-(4x-3)\}\,dx-\text{Area of }\Delta AOD$

$=\int_0^2(x^2-4x+4)\,dx-\frac{1}{2}\times\frac{3}{4}\times 3$

$=\left[\frac{(x-2)^3}{3}\right]_0^2-\frac{9}{8}=\frac{37}{24}$

28. (b)



Two curves will intersect in the Ist quadrant at $A\left(\frac{1}{R},\frac{1}{R}\right)$

$\because$ area of shaded region = 1.

$\therefore \int_0^{\frac{1}{k}}\left(\frac{\sqrt{x}}{\sqrt{k}}-kx^2\right)dx=1$

$\Rightarrow \left(\frac{1}{\sqrt{k}}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^{\frac{1}{k}}-\left(k\cdot\frac{x^3}{3}\right)_0^{\frac{1}{k}}=1$

$$\Rightarrow \frac{2}{3\sqrt{k}}\cdot\frac{1}{k^{\frac{3}{2}}}-\frac{k}{3k^3}=1$$

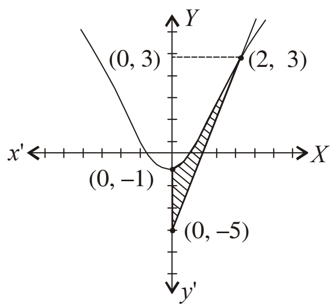
$$\Rightarrow \frac{2}{3k^2}-\frac{1}{3k^2}=1$$

$$\Rightarrow 3k^2=1$$

$$\Rightarrow k=\pm\frac{1}{\sqrt{3}}$$

$$\therefore\ k=\frac{1}{\sqrt{3}}\ (\because k>0)$$

29. **(a)**



$\because$ Curve is given as :

$y=x^2-1$

$$\Rightarrow \frac{dy}{dx}=2x$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,3)}=4$$

$\therefore$ equation of tangent at (2, 3)

$(y-3)=4(x-2)$

$\Rightarrow y=4x-5$

but $x=0$

$\Rightarrow y=-5$

Here the curve cuts Y–axis

$\therefore$ required area

$$=\frac{1}{4}\int_{-5}^{3}(y+5)dy-\int_{-1}^{3}\sqrt{y+1}\,dy$$

$$=\frac{1}{4}\left[\frac{y^2}{2}+5y\right]_{-5}^{3}\frac{-2}{3}\left[(y+1)^{3/2}\right]_{-1}^{3}$$

$$=\frac{1}{4}\left[\frac{9}{2}+15-\frac{25}{2}+25\right]$$

$$=-\frac{2}{3}\left[4^{3/2}-0\right]$$

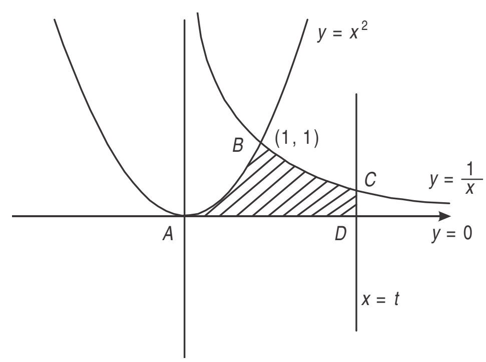
$$=\frac{32}{4}-\frac{16}{3}=\frac{8}{3}\text{ sq-units.}$$

30. **(b)** The intersection point of $y=x^2$ and $y=\frac{1}{x}$ is (1, 1)

Area bounded by the curves is the region ABCDA is given as:

$$\text{Area}=\int_0^1 x^2\,dx+\int_1^t\frac{1}{x}\,dx$$

$$=\left[\frac{x^3}{3}\right]_0^1+\left[\ln(x)\right]_1^t=\frac{1}{3}+\ln(t)$$

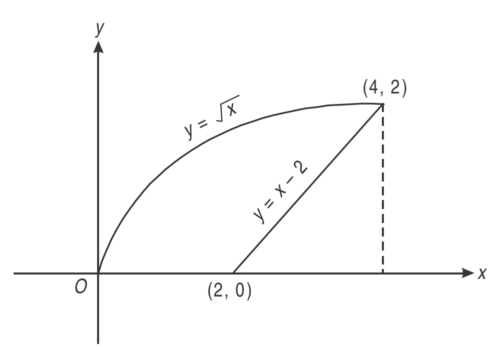


$\because$ area = 1

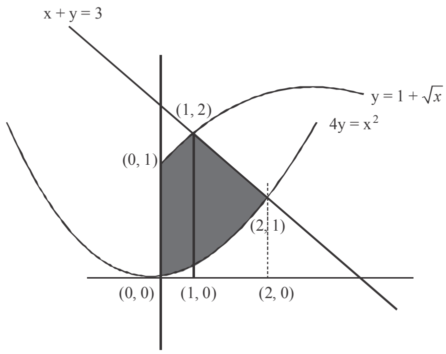
$$\Rightarrow \frac{1}{3}+\ln(t)=1\quad\Rightarrow \ln(t)=\frac{2}{3}\quad\Rightarrow t=e^{\frac{2}{3}}$$

31. **(b)** The intersection point of $y=x-2$ and $y=\sqrt{x}$ is (4, 2). The required area

$$=\int_0^4\sqrt{x}dx-\frac{1}{2}\times2\times2=\frac{16}{3}-2=\frac{10}{3}$$



32. (a)

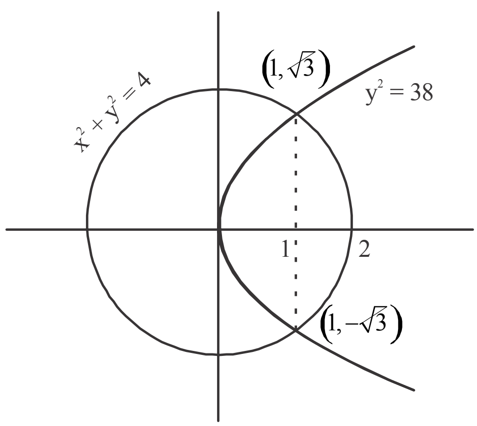


Area of shaded region

$$= \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$$

$$= [x]_0^1 + \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 + [3x]_1^2 - \left[\frac{x^2}{2}\right]_1^2 - \left[\frac{x^3}{12}\right]_0^2 = \frac{5}{2}\text{sq.units}$$

33. (d)



From the equations we get;

$x^2 + 3x - 4 = 0$

$\Rightarrow \quad (x+4)(x-1) = 0 \Rightarrow x = -4, x = 1$

when $x = 1,\ y = \sqrt{3}$

$$\text{Area} = \int_0^1 \left(\int_0^1 \sqrt{3}\,.\sqrt{x}\,dx + \int_1^2 \sqrt{4-x^2}\,.\,dx \right) \times 2$$

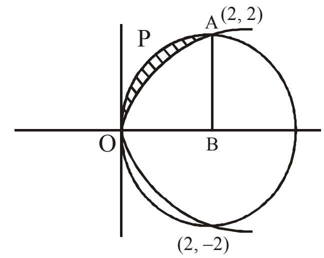
$$= \left(\sqrt{3}\left(\frac{x^{3/2}}{3/2}\right)_0^1 + \left(\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right)_1^2 \right) \times 2$$

$$= \left\{ \sqrt{3}\left(\frac{2}{3}\right) + \left\{ 2\,.\,\frac{\pi}{2} - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3}\right) \right\} \right\} \times 2$$

$$= \left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3} \right) \times 2$$

$$= \left(\frac{1}{2\sqrt{3}} + \frac{2\pi}{3} \right) \times 2 = \frac{1}{\sqrt{3}} + \frac{4\pi}{3}$$

34. (d)

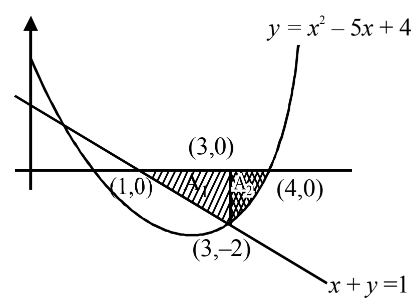


Points of intersection of the two curves are (0, 0), (2, 2) and (2, –2)

Area = Area (OPAB) – area under parabola (0 to 2)

$$= \frac{\pi \times (2)^2}{4} - \int_0^2 \sqrt{2}\sqrt{x}\ dx = \pi - \frac{8}{3}$$

35. (a)

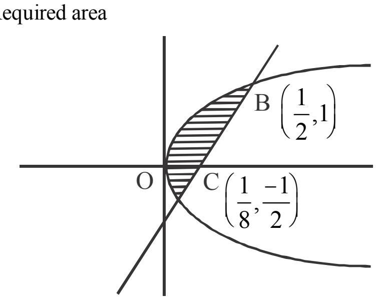


Required area $= A_1 + A_2$

$$= \frac{1}{2} \times 2 \times 2 + \left| \int_3^4 (x^2 - 5x + 4)\,dx \right|$$

$$= 2 + \frac{7}{6} = \frac{19}{6} \text{ sq. units}$$

36. (b) Required area



$$= \int_{-1/2}^{1} \frac{y+1}{4}dy - \int_{-1/2}^{1} \frac{y^2}{2}dy$$

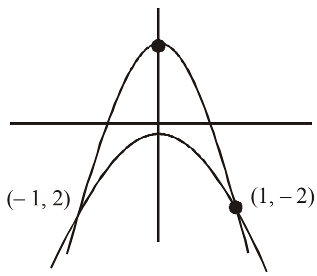
$$= \frac{1}{4}\left[\frac{y^2}{2}+y\right]_{-1/2}^{1} - \frac{1}{2}\left[\frac{y^3}{3}\right]_{-1/2}^{1}$$

$$= \frac{1}{4}\left[\frac{3}{2}+\frac{3}{8}\right] - \frac{9}{48} = \frac{15}{32} - \frac{9}{48} = \frac{27}{96} = \frac{9}{32}$$

37. **(c)** Solving

$y + 2x^2 = 0$

$y + 3x^2 = 1$



Point of intersection $(1, -2)$ and $(-1, -2)$

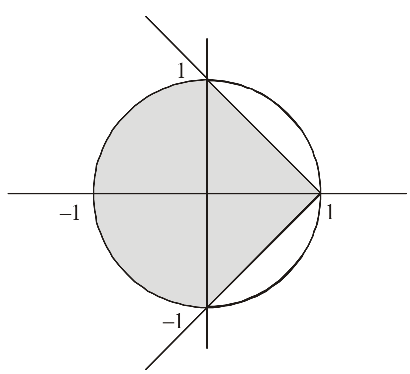
$$\text{Area} = 2\int_0^1 \left(\left(1-3x^2\right)-\left(-2x^2\right)\right)dx$$

$$2\int_0^1 \left(1-x^2\right)dx = 2\left(x - \frac{x^3}{3}\right)_0^1 = \frac{4}{3}$$

$= 15 - 6 = 9$ sq units

38. **(c)** Given curves are $x^2 + y^2 = 1$ and $y^2 = 1 - x$.

Intersecting points are $x = 0, 1$



Area of shaded portion is the required area.

So, Required Area = Area of semi-circle + Area bounded by parabola

$$= \frac{\pi r^2}{2} + 2\int_0^1 \sqrt{1-x}\,dx = \frac{\pi}{2} + 2\int_0^1 \sqrt{1-x}\,dx$$

($\because$ radius of circle = 1)

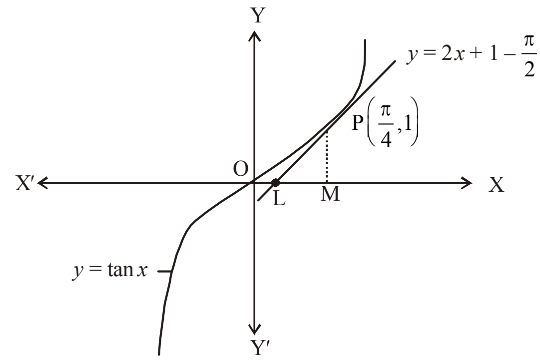
$$= \frac{\pi}{2} + 2\left[\frac{(1-x)^{3/2}}{-3/2}\right]_0^1 = \frac{\pi}{2} - \frac{4}{3}(-1) = \frac{\pi}{2} + \frac{4}{3} \text{ sq. unit}$$

39. **(a)** The given curve is $y = \tan x$ ...(1)

when $x = \frac{\pi}{4}$, $y = 1$

Equation of tangent at P is

$$y - 1 = \left(\sec^2\frac{\pi}{4}\right)\left(x - \frac{\pi}{4}\right)$$



or $y = 2x + 1 - \frac{\pi}{2}$ ...(2)

Area of shaded region

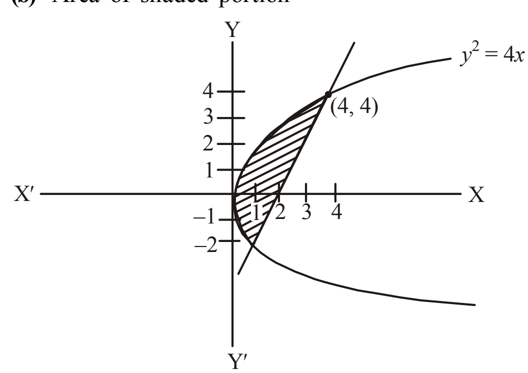
= area of OPMO – *ar* (ΔPLM)

$$= \int_0^{\frac{\pi}{4}} \tan x\,dx - \frac{1}{2}(\text{OM} - \text{OL})\text{PM}$$

$$= [\log \sec x]_0^{\frac{\pi}{4}} - \frac{1}{2}\left\{\frac{\pi}{4} - \frac{\pi-2}{4}\right\} \times 1$$

$$= \frac{1}{2}\left[\log 2 - \frac{1}{2}\right] \text{ sq unit}$$

40. **(b)** Area of shaded portion



$$= \left|\int_2^4 \left(\frac{y+4}{2}\right)dy\right| - \left|\int_{-2}^4 \frac{y^2}{4}dy\right|$$

$$= \left|\frac{1}{2}\left[\frac{y^2}{2} + 4y\right]_{-2}^4\right| - \left|\frac{1}{4}\left[\frac{y^3}{3}\right]_{-2}^4\right|$$

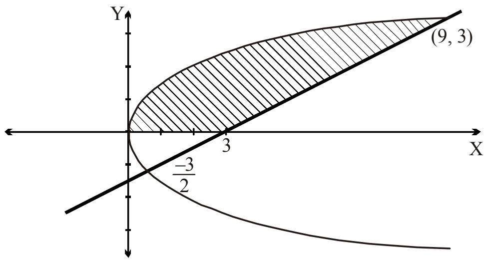
$$= \left|\frac{1}{2}[\{8+16\} - \{2-8\}]\right| - \left|\frac{1}{4}\left\{\frac{64}{3} + \frac{8}{3}\right\}\right| = 9$$

41. **(a)** Given curves are

$y=\sqrt{x}$ ...(1)

and $2y-x+3=0$ ...(2)

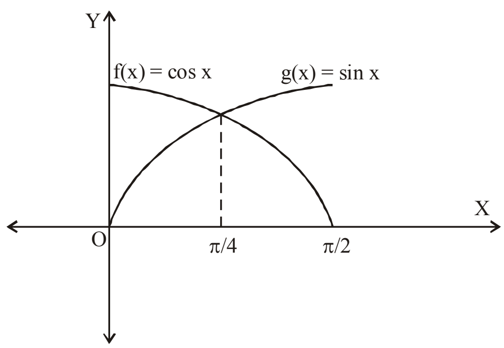
On solving both we get $y=-1, 3$



Required area $=\int_0^3\left\{(2y+3)-y^2\right\}dy$

$=\left[y^2+3y-\frac{y^3}{3}\right]_0^3=9.$

42. **(b)** $y=|\cos x-\sin x|$



Required area $=2\int_0^{\pi/4}(\cos x-\sin x)\,dx$

$=2\left[\sin x+\cos x\right]_0^{\pi/4}$

$=2\left[\frac{2}{\sqrt{2}}-1\right]=(2\sqrt{2}-2)$ sq. units

43. **(d)** Required area $=\int_{y=1}^{4}\sqrt{\frac{y}{9}}\,dy$

$=\frac{1}{3}\int_{y=1}^{4}y^{1/2}\,dy=\frac{1}{3}\times\frac{2}{3}(y^{3/2})\Big|_1^4$

$=\frac{2}{9}[(4^{1/2})^3-(1^{1/2})^3]=\frac{2}{9}[8-1]$

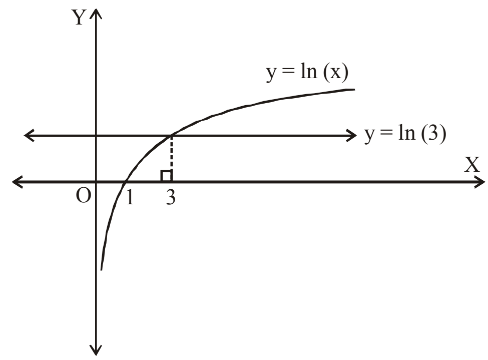
$=\frac{2}{9}\times 7=\frac{14}{9}$ sq. units.

44. **(d)** To find the point of intersection of curves $y=\ln(x)$ and $y=\ln(3)$, put $\ln(x)=\ln(3)$

$\Rightarrow \ln(x)-\ln(3)=0$

$\Rightarrow \ln(x)-\ln(3)=\ln(1)$

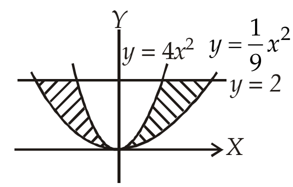
$\Rightarrow \frac{x}{3}=1, \Rightarrow x=3$



Required area $=\int_0^3 \ln(3)\,dx-\int_1^3\ln(x)\,dx$

$=[x\ln(3)]_0^3-[x\ln(x)-x]_1^3=2$

45. **(c)**



Required area $=2\int_0^2\left(\sqrt{9y}-\sqrt{\frac{y}{4}}\right)dy$

$=2\int_0^2\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy=2\left[\frac{2}{3}\times 3.y^{\frac{3}{2}}-\frac{1}{2}\times\frac{2}{3}.y^{\frac{3}{2}}\right]_0^2$

$=2\left[2y^{\frac{3}{2}}-\frac{1}{3}y^{\frac{3}{2}}\right]_0^2=2\times\left[\frac{5}{3}y^{\frac{3}{2}}\right]_0^2$

$=2.\frac{5}{3}2\sqrt{2}=\frac{20\sqrt{2}}{3}$

46. **(b)** Intersecting points are $x=1, 4$

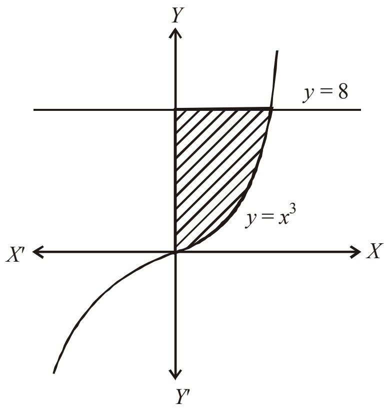
$\therefore$ Required area $=\int_1^4\left[2\sqrt{x}-\left(\frac{2x+4}{3}\right)\right]dx$

$=\frac{2x^{3/2}}{3/2}\Big|_1^4-\frac{2x^2}{3\times 2}\Big|_1^4-\frac{4}{3}x\Big|_1^4$

$=\frac{4}{3}\left(4^{3/2}-1^{3/2}\right)-\frac{1}{3}(16-1)-\left[\frac{4}{3}(4)-\frac{4}{3}\right]$

$= \frac{4}{3}(7) - 5 - 4 = \frac{28}{3} - 9 = \frac{28-27}{3} = \frac{1}{3}$

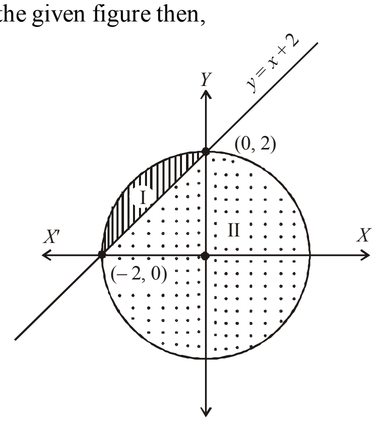
47. **(b)** Required Area $= \int_{y=0}^{8} y^{1/3} dy$



$$= \left. \frac{y^{\frac{1}{3}+1}}{\frac{1}{3}+1} \right|_0^8 = \frac{3}{4}\left(y^{4/3}\right)\Big|_0^8$$

$$= \frac{3}{4}\left[(8)^{4/3} - 0\right] = \frac{3}{4}\left[2^4\right] = \frac{3}{4} \times 16 = 12 \text{ sq. unit.}$$

48. **(d)** Let I be the smaller portion and II be the greater portion of the given figure then,



Area of $I = \int_{-2}^{0} \left[\sqrt{4-x^2} - (x+2)\right] dx$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{0} - \left[\frac{x^2}{2} + 2x\right]_{-2}^{0}$$

$$= \left[2\sin^{-1}(-1)\right] - \left[-\frac{4}{2} + 4\right] = 2 \times \frac{\pi}{2} - 2 = \pi - 2$$

Now, area of II = Area of circle – area of I.

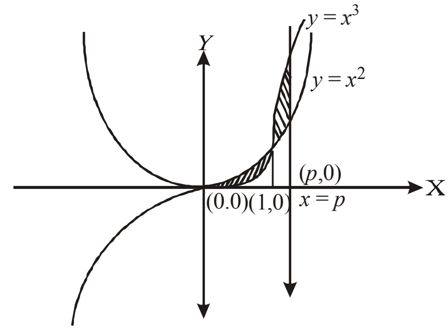
$= 4\pi - (\pi - 2) = 3\pi + 2$

Hence, required ratio $= \frac{\text{area of I}}{\text{area of II}} = \frac{\pi - 2}{3\pi + 2}$

49. **(d)** Given curves are $y = x^2$ and $y = x^3$

Also, $x = 0$ and $x = p, p > 1$

Now, intersecting point is (1, 1)



Required Area $= \int_0^1 \left(x^2 - x^3\right) dx + \int_1^p \left(x^3 - x^2\right) dx$

$$\frac{1}{6} = \left. \frac{x^3}{3} - \frac{x^4}{4} \right|_0^1 + \left. \frac{x^4}{4} - \frac{x^3}{3} \right|_1^p$$

$$\Rightarrow \frac{1}{6} = \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{p^4}{4} - \frac{p^3}{3} - \frac{1}{4} + \frac{1}{3}\right)$$

$$\Rightarrow \frac{1}{6} - \frac{1}{3} + \frac{1}{4} + \frac{1}{4} - \frac{1}{3} = \frac{3p^4 - 4p^3}{12}$$

$$\Rightarrow \frac{p^3(3p-4)}{12} = 0 \Rightarrow p^3(3p-4) = 0$$

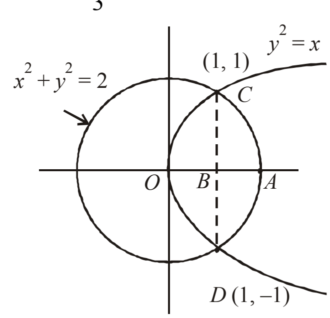
$$\Rightarrow p = 0 \text{ or } \frac{4}{3}$$

Since, it is given that $p > 1$

$\therefore p$ can not be zero.

Hence, $p = \frac{4}{3}$

50. **(b)**



Area of circle $= \pi\left(\sqrt{2}\right)^2 = 2\pi$

Area of $OCADO = 2\{\text{Area of } OCAO\}$

$= 2\ \{\text{area of } OCB + \text{area of } BCA\}$

$$= 2\int_0^1 y_p dx + 2\int_1^{\sqrt{2}} y_c dx$$

where $y_p = \sqrt{x}$ and $y_c = \sqrt{2-x^2}$

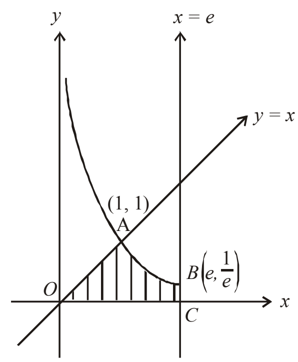
$$\therefore \text{Required Area} = 2\int_0^1 \sqrt{x}\ dx + 2\int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx$$

$$= 2\left[\frac{2}{3}.1-0\right] + 2\left[\frac{x\sqrt{2-x^2}}{2} + \sin^{-1}\frac{x}{\sqrt{2}}\right]_1^{\sqrt{2}}$$

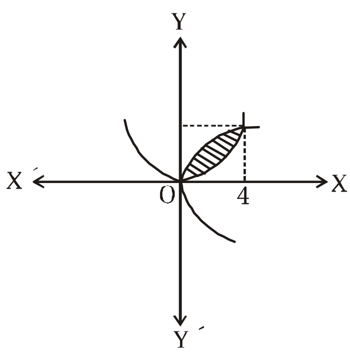
$$= \frac{4}{3} + 2\left\{\frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2}\right\} = \frac{4}{3} + 2\left\{\frac{\pi}{4} - \frac{1}{2}\right\} = \frac{3\pi+2}{6}$$

$$\text{Bigger area} = 2\pi - \frac{3\pi+2}{6} = \frac{9\pi-2}{6}$$

$$\therefore \text{Required Ratio} = \frac{9\pi-2}{3\pi+2} \text{ i.e., } 9\pi - 2 : 3\pi + 2$$



51. (b)



$$\text{Required area} = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$$

$$= \left[2\left(\frac{x^{3/2}}{3/2}\right) - \frac{x^3}{12}\right]_0^4$$

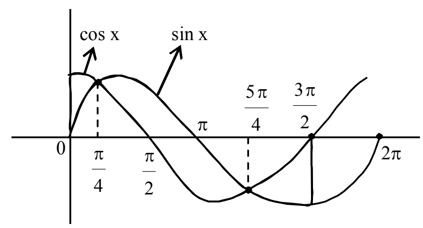
$$= \frac{4}{3} \times 8 - \frac{64}{12} = \frac{32}{3} - 16 = \frac{16}{3} \text{ sq. units}$$

52. (b) Area of required region $AOCBO$

$$= \int_0^1 x dx + \int_1^e \frac{1}{x} dx = \left[\frac{x^2}{2}\right]_0^1 + [\log x]_1^e$$

$$= \frac{1}{2} + 1 = \frac{3}{2} \text{ sq. units}$$

53. (d)



Area above x-axis = Area below x-axis

$\therefore$ Required area

$$= 2\left[\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} \sin x dx - \int_{\pi/4}^{\pi/2} \cos x dx\right]$$

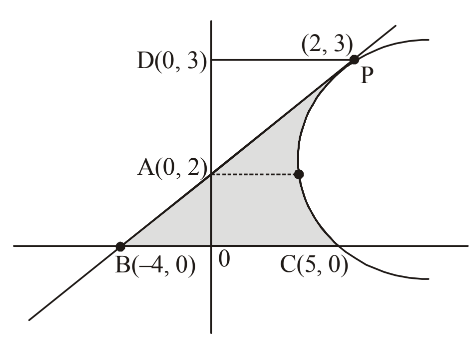
$$= 2\left[(\sin x + \cos x)_0^{\pi/4} + (-\cos x)_{\pi/4}^{\pi} - (\sin x)_{\pi/4}^{\pi/2}\right]$$

$$= 2\left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0+1) + \left(1 + \frac{1}{\sqrt{2}}\right) - \left(1 - \frac{1}{\sqrt{2}}\right)\right]$$

$$= 2\left[\sqrt{2} - 1 + 1 + \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}}\right]$$

$$= 2\left[\sqrt{2} + \sqrt{2} - 1\right] = 4\sqrt{2} - 2$$

54. (b)



For slope of tangents at (2, 3)

$(y-2)^2 = x - 1$

$2(y-2)\frac{dy}{dx}=1 \Rightarrow \frac{dy}{dx}=\frac{1}{2(y-2)}$

$m=\left(\frac{dy}{dx}\right)_{(2,3)}=\frac{1}{2(3-2)}=\frac{1}{2}$

Equation of tangent

$y-3=\frac{1}{2}(x-2)$

$\Rightarrow x-2y+4=0$ ...(i)

The given parabola is $(y-2)^2=x-1$ ...(ii)

vertex (1, 2) and it meets x-axis at (5, 0)

Then required area = Ar ΔBOA + Ar (OCPD) – Ar (ΔAPD)

$=\frac{1}{2}\times 4\times 2+\int_0^3 x\,dy-\frac{1}{2}\times 2\times 1$

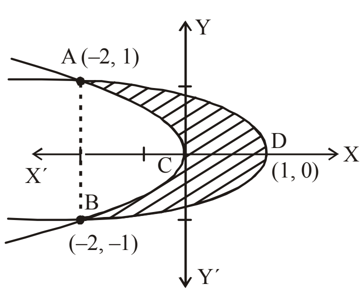
$=3+\int_0^3 (y-2)^2+1\,dy=3+\left[\frac{(y-2)^3}{3}+y\right]_0^3$

$=3+\left[\frac{1}{3}+3+\frac{8}{3}\right]=3+6=9$ sq. units

55. **(d)** Given $x+2y^2=0 \Rightarrow y^2=-\frac{x}{2}$

and $x+3y^2=1 \Rightarrow y^2=-\frac{1}{3}(x-1)$

On solving these two equations we get the points of intersection as (–2, 1), (–2, –1)



The required area is ACBDA, given by

$$A=2\left\{\int_{-2}^{1}\frac{1}{\sqrt{3}}\sqrt{1-x}\,dx-\frac{1}{\sqrt{2}}\int_{-2}^{0}\sqrt{-x}\,dx\right\}$$

$$\Rightarrow 2\left\{\frac{1}{\sqrt{3}}\left[\frac{2}{3}(1-x)^{3/2}\right]_{-2}^{1}-\frac{1}{\sqrt{2}}\left[\frac{2}{3}(-x)^{3/2}\right]_{-2}^{0}\right\}$$

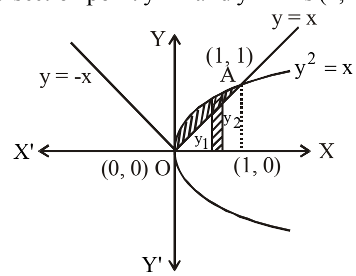
$$\Rightarrow 2\left\{\left[-\frac{1}{\sqrt{3}}\times\frac{2}{3}\left(0-3^{3/2}\right)\right]-\left[\frac{-1}{\sqrt{2}}\times\frac{2}{3}\left(0-2^{3/2}\right)\right]\right\}$$

$$\Rightarrow 2\left\{\frac{2}{\cancel{3}\sqrt{\cancel{3}}}\times\cancel{3}\sqrt{\cancel{3}}-\frac{1}{\sqrt{\cancel{2}}}\times\frac{2}{3}\cdot 2\sqrt{\cancel{2}}\right\}$$

$$\Rightarrow 2\left\{2-\frac{4}{3}\right\}=2\left\{\frac{6-4}{3}\right\}=\frac{4}{3}\text{ sq. units}$$

56. **(a)** It is clear from the figure, area lies between $y^2=x$ and $y=x$

Intersection point $y=x$ and $y^2=x$ is (1, 1)

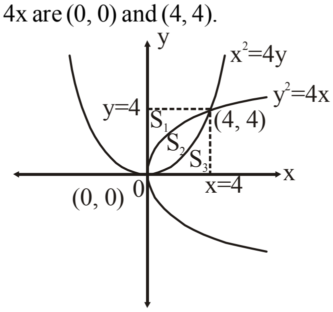


$\therefore$ Required area $=\int_0^1 (y_2-y_1)dx$

$=\int_0^1(\sqrt{x}-x)dx=\left[\frac{x^{3/2}}{3/2}-\frac{x^2}{2}\right]_0^1$

$=\frac{2}{3}\left[x^{3/2}\right]_0^1-\frac{1}{2}\left[x^2\right]_0^1=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$

57. **(d)** On solving, we get intersection points of $x^2=4y$ and $y^2=4x$ are (0, 0) and (4, 4).



By symmetry, we observe

$S_1=S_3=\int_0^4 y\,dx$

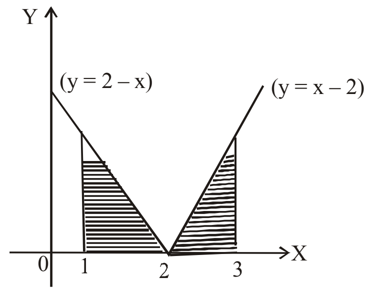
$=\int_0^4\frac{x^2}{4}dx=\left[\frac{x^3}{12}\right]_0^4=\frac{16}{3}$ sq. units

Also $S_2=\int_0^4\left(2\sqrt{x}-\frac{x^2}{4}\right)dx=\left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{x^3}{12}\right]_0^4$

$=\frac{4}{3}\times 8-\frac{16}{3}=\frac{16}{3}$ sq. units

$\therefore S_1:S_2:S_3=1:1:1$

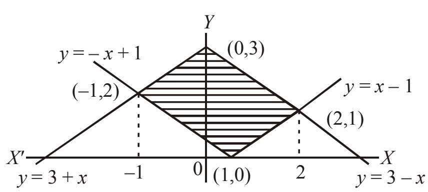
58. **(d)**



Required Area

$$A = 2\int_2^3 (x-2)dx = 2\left[\frac{x^2}{2} - 2x\right]_2^3 = 1$$

59. **(d)** Intersection point of $y = x - 1$ and $y = 3 - x$ is (2, 1) and eqns. $y = -x + 1$ and $y = 3 + x$ is (–1, 2)



$$A = \int_{-1}^{0} \{(3+x) - (-x+1)\}\, dx +$$

$$\int_0^1 \{(3-x) - (-x+1)\}\, dx + \int_1^2 \{(3-x) - (x-1)\}\, dx$$

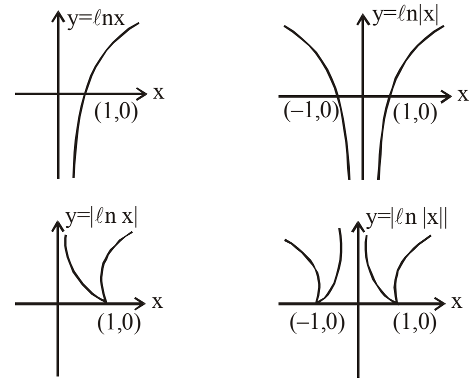
$$= \int_{-1}^{0} (2+2x)dx + \int_0^1 2dx + \int_1^2 (4-2x)dx$$

$$= \left[2x + x^2\right]_{-1}^{0} + \left[2x\right]_0^1 + \left[4x - x^2\right]_1^2$$

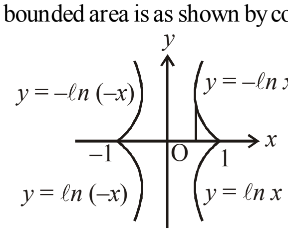
$$= 0 - (-2+1) + (2-0) + (8-4) - (4-1)$$

$$= 1 + 2 + 4 - 3 = 4\ sq.\ \text{units}$$

60. **(a)** Separate graph of each curve



[Note: Graph of $y = |f(x)|$ can be obtained from the graph of the curve $y = f(x)$ by drawing the mirror image of the portion of the graph below x-axis, with respect to x-axis.
Hence the bounded area is as shown by combined all figure.



Required area $= 4\int_0^1 (-\ell nx)dx$

$= -4\left[x\ \ell n\ x - x\right]_0^1 = 4$ sq. units